## Pre-Calculus Fall Final Exam 2014

Name: $\qquad$ Pd: $\qquad$ Date: $\qquad$

## Part 1: No calculator

## Question: 1



## Question: 2

Which of the following is true about symmetry of an equation?
A) Graph is symmetric with respect to the $x$-axis if for every point ( $x, y$ ) on the graph, the point $(-x, y)$ is also on the graph.
B) Graph is symmetric with respect to the $\mathbf{y}$-axis if for every point ( $x, y$ ) on the graph, the point $(-x,-y)$ is also on the graph.
C) Graph is symmetric with respect to the origin if for every point $(x, y)$ on the graph, the point $(-x,-y)$ is also on the graph.
D) Graph is symmetric with respect to the $y$-axis if for every point $(x, y)$ on the graph, the point $(x,-y)$ is also on the graph.

## Question: 3

Which of the following is one of the roots of the equation below?

$$
\frac{2}{x+2}+\frac{3}{x-1}=\frac{-8}{5}
$$

A) 4
B) $\frac{1}{8}$
C) -8
D) $-\frac{1}{8}$

## Question: 4

Solve for $x$ :

$$
4^{5 x-1}=\sqrt[3]{32}
$$

A) $\frac{11}{10}$
B) $\frac{1}{30}$
C) $\frac{11}{30}$
D) $-\frac{11}{30}$

## Question: 5

Determine the Period of the trigonometric function below.

$$
f(x)=\frac{1}{2} \operatorname{Sin}\left(\frac{3}{2} x-\pi\right)
$$

A) $\frac{4}{3} \pi$
B) $\frac{2}{3} \pi$
C) $\frac{3}{4} \pi$
D) $\frac{3}{2} \pi$

## Question: 6

Solve for x , from the log equation below.

$$
\log _{6}(35 x+6)-\log _{6} x=2
$$

A) $\frac{42}{36}$
B) 6
C) -6
D) Not here

## Question: 7

Evaluate the following trigonometric expression:

$$
\operatorname{Sin} 80^{\circ} \operatorname{Sin} 50^{\circ}+\operatorname{Sin} 10^{\circ} \operatorname{Sin} 40^{\circ}
$$

A) $-\frac{\sqrt{3}}{2}$
B) $\frac{1}{2}$
C) $\frac{\sqrt{2}}{2}$
D) $\frac{\sqrt{3}}{2}$

## Question: 8

A rectangle is inscribed in a semicircle of radius 2 as shown below. Let $P=(x, y)$ be the point on quadrant 1 that is a vertex of the rectangle and is on the circle $\left(y=\sqrt{4-x^{2}}\right)$.


Express the area of the rectangle as a function of x .
A) $x \sqrt{4-x^{2}}$
B) $2 x \sqrt{4-x^{2}}$
C) $\sqrt{8 x-2 x^{3}}$
D) $-x \sqrt{4-x^{2}}$

## Question: 9

For which of the following graphs of $f$ does
$f(\mathrm{x})=f(-\mathrm{x})$ for all values of x shown?
(A)

(C)

(E)

(B)

(D)


## Question: 10

Solve for $y$ without the use of a calculator.
If $4^{y}=8^{x}$ and $3^{x}=2\left(3^{y}\right)$
A) $\frac{3 \ln 2}{\ln 3}$
B) $\frac{-2 \ln 2}{\ln 3}$
C) $\frac{-3 \ln 2}{\ln 3}$
D) $\frac{2 \ln 2}{\ln 3}$

## Question: 11

The terminal side of an angle $\theta$, in standard position passes through the point $(7,-24)$. What is $\csc \theta$ ?
a. $-\frac{24}{25}$
b. $-\frac{25}{24}$
c. $\frac{24}{25}$
d. $-\frac{25}{7}$
e. $-\frac{24}{7}$

## Question: 12** Calculator allowed

An airplane flies from city $A$ to city $B$, a distance of 150 miles, and then turns through an angle of $40^{\circ}$ and heads to city C , as shown in the figure below.


If the distance between city A and city C is 300 miles, how far is it from city B to city C?
A) $\mathbf{1 6 9 . 1 8 \text { miles }}$
B) $140^{\circ}$
C) 200 miles
D) 185.23 miles

## Question: 13

Find the exact value of the expression below.

$$
\operatorname{Sin}^{2}\left(20^{0}\right)+\frac{1}{\operatorname{Sec}^{2}\left(20^{0}\right)}
$$

A. 0
B. 20
C. -1
D. 1
E. None

## Question: 14

From the double-angle formula below, derive a half-angle formula.

$$
\operatorname{Cos}^{2} \alpha=\frac{1+\cos (2 \alpha)}{2}
$$

A) $\operatorname{Sin} \frac{\alpha}{2}= \pm \sqrt{\frac{1+\cos \alpha}{2}}$
B) $\operatorname{Cos} \frac{\alpha}{2}= \pm \sqrt{\frac{1+\operatorname{Sin} \alpha}{2}}$
C) $\operatorname{Cos} \frac{\alpha}{2}= \pm \sqrt{\frac{1+\cos \alpha}{2}}$
D) $\operatorname{Cos} \frac{\alpha}{2}= \pm \sqrt{\frac{1-\cos \alpha}{2}}$

## Question: 15

Which statement is completely true about the cubic function $f(x)=x^{3}$
A) $f$ is even, and always increasing.
B) $f$ is odd and symmetric about the $x$-axis
C) $f$ is odd, symmetric about the $y$-axis, and has no local minimum
D) $f$ is odd, symmetric about the origin, and has no local maximum

## Question: 16

Form the difference quotient for $f(x)=\frac{1}{5 x+7}$ and evaluate it at $x=-3$.
a. $\frac{-1}{8}$
b. $\frac{-5 h}{64-40 h}$
c. $\frac{-5}{40 h-64}$
d. $\frac{5}{40 h-64}$
-5

## Question: 17

If $\theta$ is an acute angle and $\sin (\theta)=\frac{3}{5}$, Evaluate $\operatorname{Sin}(2 \theta)+\operatorname{Cos}(2 \theta)$
A) $\frac{41}{25}$
B) ) $\frac{31}{25}$
C) ) $\frac{17}{25}$
D) ) $\frac{17}{5}$

## Question: 18

Evaluate the discriminant of the quadratic equation below.

$$
y=2 x^{2}-3 x+4
$$

A) -23
B) -41
C) 41
D) Not here

## Question: 19

$$
\text { What is the domain for } \mathrm{f}(\mathrm{x})=\frac{1}{\sqrt{5^{2 \mathrm{x}-1}-125}} \text { ? }
$$

A. $(-\infty, \infty)$
D. $(2, \infty)$
B. $(-\infty, 2) \cup(2, \infty)$
E. None of these
C. $\{3,4\}$

## Question: $\mathbf{2 0}$

Find the exact value of the expression below.
$6 \cos \left(\frac{3 \pi}{4}\right)+2 \tan \left(-\frac{\pi}{3}\right)$
A. $3 \sqrt{2}-2 \sqrt{3}$
B. $-3 \sqrt{2}-2 \sqrt{3}$
C. $-3 \sqrt{2}+2 \sqrt{3}$
D. None

## Question: 21

Evaluate the following log:

$$
\log _{8} \sqrt{2}
$$

A) $\frac{3}{2}$
B) $\frac{2}{3}$
C) $\frac{1}{6}$
D) Not here

## Question: $\mathbf{2 2}$

Evaluate the Axis of symmetry of the quadratic equation below.

$$
y=2 x^{2}-3 x+4
$$

A) $x=\frac{4}{3}$
B) $x=\frac{-3}{8}$
C) $x=\frac{-3}{4}$
D) $x=\frac{3}{4}$

## Question: 23

Which of the following can be the properties of the given polynomial?

A) $n \geq 2$ even, $a_{n}<0$
B) $\mathrm{n} \geq 3$ odd, $\mathrm{a}_{\mathrm{n}}>0$
C) $n \geq 3$ odd, $a_{n}<0$
D) $n \geq 2$ even, $a_{n}>0$

## Question: 24

Which answer choice is an incorrect statement about functions and their inverses?

A The domain values of $f(x)$ are equal to the range values of $f^{-1}(x)$ and the domain values of $f^{-1}(x)$ are equal to the range values of $f(x)$.

B The graphs of $f(x)$ and $f^{-1}(x)$ are reflections of each other over the line $y=x$.
C To determine the equation for $f^{-1}(x)$, first determine the equation for $f\left(\frac{1}{x}\right)$ and then solve that equation for $x$.

D The point $(b, a)$ is on the graph of $f^{-1}(x)$ when the point $(a, b)$ is on the graph of $f(x)$.

## Question: 25

Which of the following can be a representation of the Law of cosine, for a triangle with sides $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and opposite angles $\alpha, \beta, \gamma$, respectively?
A) $c^{2}=a^{2}+b^{2}-2 a b \cos (\gamma)$
B) $a^{2}=c^{2}+b^{2}-2 a b \cos (\gamma)$
C) $c^{2}=a^{2}+b^{2}-2 a b \cos (\alpha)$
D) $c^{2}=a^{2}+b^{2}-2 a b \cos (\beta)$

## Question: 26

## (From Free Response \# 3)

Use completing the square method to write the quadratic function below in the form $f(x)=a(x-h)^{2}+k$, and give the value of $k$.

$$
y=2 x^{2}+3 x-4
$$

A) $\frac{41}{8}$
B) $-\frac{41}{8}$
C) $\frac{23}{8}$
D) $-\frac{23}{8}$

## Question: 27

Determine limit ${ }_{h \rightarrow 0} \frac{[f(x+h)-f(x)]}{h}$, if $f(x)=2 x^{2}-3 x$
A. $4 x-3+h$
B. $4 \mathrm{x}-3$
C. $4 x+2 h-3 x$
D. $4 x-3+2 h$
E. None

## Question: 28

Determine the Period of the trigonometric function below.

$$
f(x)=\frac{1}{2} \operatorname{Sin}\left(\frac{3}{2} x-\pi\right)
$$

A) $\frac{4}{3} \pi$
B) $\frac{2}{3} \pi$
C) $\frac{3}{4} \pi$
D) $\frac{3}{2} \pi$

## Question: 29

If $f(x)=4^{\frac{x-8}{2}}$ and $g(x)=3^{2-x}$ determine $f \circ g(x)$ :
A) $4^{\frac{3(2-x)-8}{2}}$
B) $4^{\frac{9-8\left(3^{x}\right)}{6^{x}}}$
C) $4^{\frac{9-8\left(3^{x}\right)}{2\left(3^{x}\right)}}$
D) $2^{\frac{9-8\left(3^{x}\right)}{4\left(3^{x}\right)}}$

## Question: 30

If $1 / 3 \log _{3} x=2 \log _{3} 2$, then $x=$ ?
A. 8
B. $4 / 3$
C. 64
D. 12

## Question: 31

Determine if the statement below is true or false about inverse cosine in the first and second quadrants.
" $y=\cos ^{-1}(x)$ means $x=\cos y$, where $-1 \leq x \leq 1$ and $0 \leq y \leq \pi "$
A) True
B) Even
C) False
D) Odd

## Question: 32

Determine $\frac{[f(x+h)-f(x)]}{h}$, if $f(x)=2 x^{2}-3 x$
A. $4 x-3+h$
B. $4 x-3$
C. $4 x+2 h-3 x$
D. $4 x-3+2 h$
E. None

## Question: 33



For the polynomial above, does the function have any zero with multiplicity greater than 1 .
A) No, it does not.
B) Yes, it does
C) Not enough information

## Question: 34

Find the exact value of the expression below.
$\operatorname{Sin}\left(-40^{\circ}\right) \operatorname{Csc}\left(40^{\circ}\right)$
A. $1 / 2$
B. -40
C. -1
D. 1
E. None

## Question: 35

$$
\text { If } \log _{a}\left(3^{a}\right)=\frac{a}{2} \text {, then } \mathrm{a}=\text { ? }
$$

A) 8
B) 2
C) 27
D) 9

## Question: 36

Find the exact value of the trigonometric function below if $\operatorname{Sin} \theta=\frac{4}{5}$, for $0^{\circ} \leq \theta \leq 90^{\circ}$ $\tan \theta=?$
A. $\frac{4}{3}$
B. $\frac{3}{5}$
C. 1
D. $3 / 4$
E. None

## Question: 37

## From Free response \#1:

Is the degree of the polynomial even or odd?
A) Even
B) Odd
C) Neither even nor odd

## Question: 38

If $3 \log _{2}(x-1)+\log _{2} 4=5$, then $x=$ ?
A. 3
B. 1
C. 8
D. None

## Question: 39

Converting $135^{\circ}$ into radians will yield?
A) $\frac{2}{3} \pi$
B) $\frac{1}{3} \pi$
C) $\frac{3}{4} \pi$
D) $\frac{3}{2} \pi$

## Question: 40

Evaluate the expression below:

$$
\log _{3} 81+\log _{5} 125
$$

A) 12
B) 14
C) 8
D) 7

## Question: 12** Calculator allowed

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If the distance between city $A$ and city $C$ is 300 miles, how far is it from city $B$ to city C?
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B) $140^{\circ}$
C) 200 miles
D) 185.23 miles

## Free Response Problem \#1


A) Is the degree of the polynomial above even or odd?
B) Is the leading coefficient of the polynomial above positive or negative?
C) Is the function above even, odd, or neither?
D) Why is $x^{2}$ necessarily a factor of the polynomial?

## Free response Problem \#2

Solve for x :

$$
3 \cdot 2^{x}-30 \cdot 2^{0.5 x}+48=0
$$

## Free response Problem \#3

Use completing the square method to write the quadratic function below in the form of: $f(x)=a(x-h)^{2}+k$,

$$
y=2 x^{2}+3 x-4
$$

## Free response Problem \#4

For the given function $f(x)=3 x^{2}+5 x-4$,
a) Determine the expression for the difference quotient for $f$, leave answer in simplest form. $\frac{f(x+h)-f(x)}{h}$
b) Evaluate the limit of the difference quotient as h approaches zero.

$$
\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}
$$

